Using Inductive Reasoning to Make Conjectures

Find the next item in each pattern.

1. 2, 4, 6, 8, . . .
3. fall, winter, spring, . . .

4. When several examples form a pattern and you assume the pattern will continue, you are applying _____________.
5. A statement you believe to be ____________ based on inductive reasoning is called a conjecture.

For Exercises 6–8, complete each conjecture by looking for a pattern in the examples.

6. The sum of two odd numbers is _____________.

\[
\begin{array}{c|c|c|c|c}
3 + 5 & 13 + 3 & 1 + 1 \\
8 & 16 & 2
\end{array}
\]

7. The number of sides of a polygon that has \( n \) vertices is _____________.

8. When a tree is cut horizontally, a series of rings is visible in the stump. Make a conjecture about the number of rings and the age of the tree based on the data in the table.

<table>
<thead>
<tr>
<th>Number of Rings</th>
<th>3</th>
<th>15</th>
<th>22</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age of Tree (years)</td>
<td>3</td>
<td>15</td>
<td>22</td>
<td>60</td>
</tr>
</tbody>
</table>

9. Assume your conjecture in Exercise 8 is true. Find the number of rings in an 82-year-old oak tree. ________________

10. A counterexample shows that a conjecture is _______________.

Show that each conjecture is false by finding a counterexample.

11. For any number \( n \), \( 2n > n \).

12. Two rays having the same endpoint make an acute angle. (Sketch a counterexample.)
5. \( G'(−2.5, 4), H'(−3.5, 2), I'(−4, 4), J'(−5, 6) \)
6. \((x, y)\rightarrow (x−7, y+5)\)

Practice C
1. \( \triangle PQR \)
2. The vertex labels do not match. For the rotation, \( P \) moves to \( Q' \) and \( Q \) moves to \( P' \).
3. \( \triangle ABC \) and \( \triangle PQR \)
4. \((-x, -y)\)
5. \((y, -x)\)
6. \((-x, -y)\)
7. \((x, y)\)
8. \((x, y) \rightarrow (x+\sqrt{3}, y+1)\)
9. 120 meters

Reteach
1. translation; possible answer: \( \triangle FGH \rightarrow \triangle F'G'H' \)
2. reflection; possible answer: \( \triangle MNP \rightarrow \triangle M'N'P' \)
3. reflection; possible answer: \( \triangle WXY \rightarrow \triangle W'X'Y' \)
4. rotation; possible answer: \( \triangle ABCD \rightarrow \triangle A'B'C'D' \)
5. translation
6. reflection
7. \( X'(−4, 6), Y'(6, 5), Z'(3, 8) \)
8. \( F'(6, 9), G'(−9, 2), H'(−5, 5) \)
9. \( B'(7, 1), C'(0, 0), D'(8, 4) \)

Challenge
1. Possible answer: first, a reflection across the \( y \)-axis; then a translation 3 units right and 5 units down
2. Possible answer: first, a reflection across the line \( y = 3 \); then a translation 8 units left and 4 units down.
3. \( W'(−7, −5), X'(−3, −5), Y'(−4, −2), Z'(−6, −2) \); preimage reflected across \( x \)-axis; image translated by \((x, y) \rightarrow (x+8, y+3)\)
4. No, the coordinates could be \( W'(1, 8), X'(5, 8), Y'(4, 5), Z'(2, 5) \).

Problem Solving
1. player 3: \((x, y) \rightarrow (x+4.5, y−1)\);
player 4: \((x, y) \rightarrow (x−4, y+1)\)
2. player 3: \((-5.5, −2)\); player 4: \((4, −1.5)\)
3. \((-5, 9), \left(\frac{1}{2}, 9\right), (-1, 6), \left(-\frac{3}{2}, 6\right)\)
4. reflection across the \( y \)-axis
5. \( A'(6, 17), C'(10, 14), D' \left(-\frac{7}{2}, 14\right)\)
6. C
7. J

Reading Strategies
1. Possible answer: translation
2. Possible answer: rotation
3. reflection

LESSON 2-1

Practice A
1. 10
2. W
3. summer
4. inductive reasoning
5. true
6. even
7. \( n \)
8. The number of rings in a tree is the same as the tree’s age.
9. 82 rings
10. false
11. Possible answers: zero, any negative number
Practice B

1. 36
2. 
3. Arkansas
4. north
5. positive
6. \(n - 3\)
7. Possible answers: zero, any negative number
8.
9. One-third of the bills were counterfeit.
10. Each item, starting with the third, is the product of the two preceding items; 256, 8192.
11. The dot skips over one vertex in a clockwise direction.

Practice C

1. The pattern is the cubes of the negative integers; \(-125, -216\).
2. Each item describes the item before it (one, one one, two ones, \ldots); \(312211, 13112221\).
3. The pattern is the letters of the alphabet that are made only from straight segments; K, L.
4. First rotate the figure \(180°\). Then reflect the figure across a vertical line. Repeat.
5. true
6. false

Sample answer:
7. true
8. false
  Possible answers: \(n = 1, n = -1\)
9. Sum of angle measures = \([180(n - 2)]°\)
10. \(128.6°; 140°; 150°\)

Reteach

1. \(\frac{1}{4}\)
2. 36
3. \(15\)
4. 
5. doubled
6. \(2n\)
7. \(n + 2\)
8. Possible answer: If the lines are parallel, then they do not intersect.
9. Possible answer: If point \(N\) is between points \(A\) and \(G\), then \(AN = 2\) inches.
10. Sample answer: If \(x = 0\) and \(y = -1\), then \(x^2 < y^2\).
11. Sample answer: \(\angle ABD, \angle DBE, \angle EBC, \angle ABE, \angle DBC\)
12. Sample answer: \(m\angle 1 = 25°, m\angle 2 = 20°\)
13. true
14. 

Challenge

1. 1, 6, 15, 20, 15, 6, 1
2. Each row has 1 as the first and last number. Each of the other numbers is found by adding the two numbers that appear just above it.
3. The sum of each row of terms after the first is twice the sum of the terms in the previous row.
4. 27; 81
5. \(3^{n - 1}\)
6. Sample answer: \(x = 2\)
7. Sample answer: \(n = -2\)
8. Sample answer: All values of \(4^a - 1\) are divisible by 3.

<table>
<thead>
<tr>
<th>(a)</th>
<th>(4^a - 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>63</td>
</tr>
<tr>
<td>4</td>
<td>255</td>
</tr>
<tr>
<td>5</td>
<td>1023</td>
</tr>
</tbody>
</table>